Two-channel Sampling in Wavelet Subspaces

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ABSTRACT

In this paper, a sampling theorem is established in each V_j of MRA$\{V_j\}$ under the necessary-sufficient condition for which it holds. Also, we improve and modify the result of Chen and Itoh[2], which shows the fallacy and the unclearness in their proof. The modified result can be extended to single-channel sampling. It can be shown that a *shift sampling* using Zak transform is a special case of our single-channel sampling. Also, we derive two-channel sampling formula in V_j, and finally, two-channel sampling formula in V_j generated by two generators using Gramian analysis.

NEGLECTED POINTS IN PREVIOUS PAPER

For $\{a_k\}_k \in l^2$ and $\{b_k\}_k \in l^2$, $\mathcal{F}^*\{a_k\}\mathcal{F}^*\{b_k\} \in L^1[0, 2\pi]$ so that one can obtain only formal Fourier series expansion $\mathcal{F}^*\{a_k\}\mathcal{F}^*\{b_k\} \sim \sum_n \Big(\sum_k a_k b_{n-k}\Big) e^{-in\xi}$ where $\mathcal{F}^*\{a_k\} := \sum_k a_k e^{-ik\xi}$. By Young's inequality $\{\sum_k a_k b_{n-k}\}_n \in l^\infty$ and thus the series may not converge. This point was neglected in the proof of previous result [1]. Also, it does not hold in general $\mathcal{F}\Big(\sum c_k \phi(t-k)\Big) = \Big(\sum c_k e^{-ik\xi}\Big)\hat{\phi}(\xi)$ when $\sum c_k \phi(t-k)$ converges in $L^2(R)$ where $\{c_k\} \in l^2$ and $\phi \in L^2(R)$. A sufficient condition for $\mathcal{F}\Big(\sum c_k \phi(t-k)\Big) = \Big(\sum c_k e^{-ik\xi}\Big)\hat{\phi}(\xi)$ to hold is that $\{\phi(t-n)\}$ is a Bessel sequence. We modify these points and obtain new versions of sampling theorem in $L^2(R)$ sense in wavelet subspaces.

REFERENCES

1. W. Chen and S. Itoh, *"A sampling theorem for shift invariant subspace", IEEE Trans. Signal Processing, vol.46, Oct. 1998, pp.2822-2824.*